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# ANALYSIS OF A STRUCTURE WITH A RANDOM GEOMETRY

By

J. Lyell Sanders Jr.



Division of Engineering and Applied Physics
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## Introduction

A metal polycrystal, a nonwoven fabric such as felt, a suspension of particles of random size, shape, and location in a viscous fluid, are examples of what we mean by a structure with a random geometry. The problem of interest in connection with such structures is the determination of some average property such as the effective Young's modulus or the effective viscosity or perhaps the probability density function of these properties. Solutions satisfactory for practical purposes to a fair number of such problems have appeared in the literature, but the author knows of no instance in which some (nontrivial) average property of a random structure has been determined exactly. The difficulty is not in solving some known mathematical equation, but rather in reducing such problems to exact mathematical terms. The purpose of the present paper is to become better acquainted with the nature of the problem by means of an exact analysis of a very simple random structure.

The paper is concerned with the analysis of a structure which is essentially one-dimensional. It turns out that the analysis of this structure is much like the analysis of a Markov process. The fact that a spacial coordinate replaces the time-like independent variable of a Markov process makes for some complication as might be expected since boundary value problems are generally more complicated than initial value problems. The results obtained give no

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clear indication of how to approach the exact analysis of any physical problem but suggest that the next step should probably be to invent some two-dimensional problem sufficiently simple to be analyzed and not so complicated as a real physical problem is likely to be.

## Analysis of a Random Structure

The structure to be considered is composed of a uniform elastic rod pin-jointed to a series of elastic beams which are built-in to a rigid foundation as shown in Figure 1.

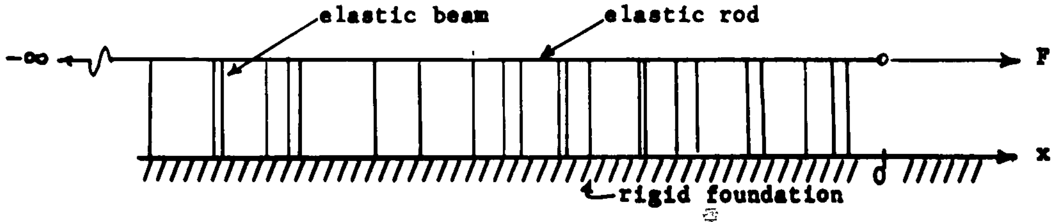


Figure 1

The beams are randomly spaced with, on the average, $N$ beams per unit length. In other words the probability of finding a beam in a length $\delta x$ is equal to $N\delta x$. If the random variable $y$ is the distance between successive beams then, as is well known in such cases, the probability density function (p.d.f.) of $y$ is given by the Poisson law $Ne^{-Ny}$. For the present suppose the rod is semi-infinite with one end at the origin. Let a force $F$ be applied to the rod at the origin and let the resulting displacement of the end be $u$. Let $\sigma = F/u$ be called the stiffness of the structure. Consider the problem of finding the p.d.f. $P(\sigma)$ of the random variable $\sigma$. First add on a beam at the origin. The stiffness of the augmented structure $\sigma'$ is

$$\sigma' = \sigma + \alpha \tag{1}$$

where $\alpha$ is the stiffness of a beam with respect to a load at its tip. The p.d.f. $\sigma'$ is thus

$$Q(\sigma') = P(\sigma)\frac{d\sigma}{d\sigma'} = P(\sigma'-\alpha) \tag{2}$$

Now add a piece of rod of length $y$. The stiffness $\sigma''$ of the augmented structure is now given by

$$\frac{1}{\sigma''} = \frac{1}{\sigma'} + \frac{1}{\kappa/y} \tag{3}$$

where $\kappa/L$ is the stiffness of a rod of length $L$. The conditional probability density function (c.p.d.f.) of $\sigma''$ given $\sigma'$ is

$$T(\sigma'';\sigma') = R(y)\left|\frac{dy}{d\sigma''}\right| = R\left(\frac{\kappa}{\sigma''} - \frac{\kappa}{\sigma'}\right)\frac{\kappa}{(\sigma'')^2} \tag{4}$$

where

$$\left.\begin{aligned} R(y) &= Ne^{-Ny} \qquad & y \geq 0 \\ &= 0 \qquad & y < 0 \end{aligned}\right\} \tag{5}$$

and the p.d.f. of $\sigma''$ is

$$\int T(\sigma'';\sigma')Q(\sigma')d\sigma' \tag{6}$$

Now the augmented structure with the beam and rod added is statistically the same as the original structure, therefore the p.d.f. of $\sigma''$ must be the same as the p.d.f. of $\sigma$ and hence

$$P(\sigma) = \frac{N\kappa}{\sigma^2}\int_{\sigma}^{\infty} e^{-N\kappa\left(\frac{1}{\sigma} - \frac{1}{\sigma'}\right)} P(\sigma'-\alpha)d\sigma' \tag{7}$$

The integral equation (7) is easily reducible to the following differential-difference equation

$$(\sigma^2 P)' - \kappa NP = -\kappa NP(\sigma-\alpha) \tag{8}$$

where a prime denotes differentiation with respect to $\sigma$. If the beams have a random stiffness $\beta$ with p.d.f. $S(\beta)$ we obtain instead of (8) the following

$$(\sigma^2 P)' - \varkappa N P = -\varkappa N \int_0^{\sigma} P(\xi) S(\sigma - \xi) d\xi \tag{9}$$

Equations (8) or (9) must be solved subject to the normalization condition

$$\int_0^{\infty} P(\sigma) d\sigma = 1 \tag{10}$$

Let $P_L(p)$ denote the Laplace transform of $P(\sigma)$. We have

$$\left.\begin{aligned} P_L(p) &= \int_0^{\infty} P(\sigma) e^{-\sigma p} d\sigma & P_L'(0) &= -\int_0^{\infty} \sigma P(\sigma) d\sigma = -\bar{\sigma} \\ P_L(0) &= 1 & P_L''(0) &= \int_0^{\infty} \sigma^2 P(\sigma) d\sigma = \overline{\sigma^2} \end{aligned}\right\} \tag{11}$$

where a bar over a quantity indicates its mean value. Taking the Laplace transform of Eq. (9) leads to

$$P_L'' - \varkappa N \frac{1 - S_L}{p} P_L = 0 \tag{12}$$

In the special case $S(\beta) = \frac{1}{\alpha} e^{-\beta/\alpha}$ for which $\bar{\beta} = \alpha$ the exact solution for $P(\sigma)$ is

$$P(\sigma) = \frac{\sqrt{\alpha \varkappa N}}{2\sigma^2 K_1\left(\sqrt{\frac{4\varkappa N}{\alpha}}\right)} e^{-\left(\frac{\varkappa N}{\sigma} + \frac{\sigma}{\alpha}\right)} \tag{13}$$

where $K_1$ is the modified Bessel function of the second kind.

It is interesting to make some comparisons with the stiffness of a similar structure with uniformly spaced beams. Let the beams be spaced uniformly at a distance $1/N$ and have a uniform stiffness $\alpha$, then the stiffness of the structure (with no beam at the finite end) is

$$\sigma = (\alpha \varkappa N + \frac{\alpha^2}{4})^{1/2} - \frac{\alpha}{2} \tag{14}$$

Now let $\alpha \to 0$, $N \to \infty$ and require $\alpha N$ to remain finite, then

$$\sigma^2 \to \alpha \varkappa N \tag{15}$$

From Eq. (12) and with $\bar{\beta} = -S_L'(0) = \alpha$

$$\overline{\sigma^2} = \lim_{p \to 0} P_L'' = \alpha \varkappa N \tag{16}$$

the same result as in (15), but in general $(\bar{\sigma})^2 \neq \overline{\sigma^2}$.

## Structure of Finite Length

The solution of the problem in the preceding section for a semi-infinite structure depended on a special device which will not work for a structure of finite length. Consider now a somewhat more complicated problem for the structure shown in Figure 2.

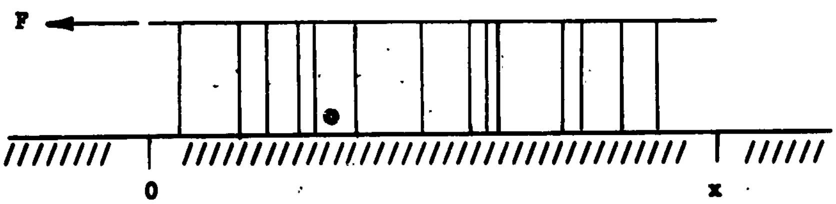


Figure 2

Let it be required to find the joint p.d.f. $R(\sigma,u,x)$ where $\sigma$ and $u$ are the stiffness and displacement at the right end. The left end of the structure is subjected to a load $F$ as shown and $u$ is taken positive to the left. This time augment the structure by an infinitesimal piece in such a way as to leave the statistical character of the structure undisturbed. Let $R_1(\sigma,u,x+\delta x)$ be the new R on condition there is no beam in the added piece of length $\delta x$, and $R_2$ be the new R on condition there is a beam in the added length. The unconditional new R is given by

$$R(\sigma,u,x+\delta x) = R_1(\sigma,u,x+\delta x)(1-N\delta x) + R_2(\sigma,u,x+\delta x)N\delta x \tag{17}$$

since we do not or do include a beam in the added piece with probabilities $1 - N\delta x$ and $N\delta x$ respectively. Expressions for $R_1$ and $R_2$ are found as follows.

1. <u>No beams added</u>. Let $\sigma'$ and $u'$ be the new displacement and stiffness. These are related to $\sigma$ and $u$ as follows

$$\left.\begin{array}{lcl} u' = u & & u = u' \\ & \text{or} & \\ \dfrac{1}{\sigma'} = \dfrac{1}{\sigma} + \dfrac{\delta x}{\kappa} & & \sigma = \dfrac{\kappa\sigma'}{\kappa-\sigma'\delta x} \end{array}\right\} \tag{18}$$

According to the calculus of probabilities

$$R_1(\sigma',u',x+\delta x)d\sigma'du' = R(\sigma,u,x)d\sigma du \tag{19}$$

where

$$d\sigma du = J\left(\frac{\sigma,u}{\sigma',u'}\right)d\sigma'du' = \left(\frac{\kappa}{\kappa-\sigma'\delta x}\right)^2 d\sigma'du' \tag{20}$$

and where $J$ is the Jacobian of the change of variable. From this it follows

$$R_1(\sigma',u',x+\delta x) = R\left(\frac{\kappa\sigma'}{\kappa-\sigma'\delta x}, u', x\right)\left(\frac{\kappa}{\kappa-\sigma'\delta x}\right)^2 \tag{21}$$

2. <u>Beam added</u>. In this case the change in stiffness due to the added rod may be neglected and we have

$$\sigma' = \sigma + \alpha \tag{22}$$

The added beam exerts a force $\alpha u'$ on the old structure and causes a displacement to the right by an amount

$$u'' = \frac{\alpha u'}{\sigma} \tag{23}$$

but

$$u' = u - u'' \tag{24}$$

therefor

$$u' = u - \frac{\alpha u'}{\sigma} \quad \text{or} \quad u = \frac{\sigma' u'}{\sigma' - \alpha} \tag{24}$$

In this case

$$J = \frac{\sigma'}{\sigma' - \alpha} \tag{25}$$

and

$$R_2(\sigma', u', x+\delta x) = R(\sigma, u, x)\frac{\sigma'}{\sigma'-\alpha} = R(\sigma'-\alpha, \frac{\sigma' u'}{\sigma'-\alpha}, x)\frac{\sigma'}{\sigma'-\alpha} \tag{26}$$

Primes can now be dropped with no loss in sense and (17) becomes

$$R(\sigma,u,x+\delta x) = R(\frac{\kappa\sigma}{\kappa-\sigma\delta x}, u, x)(\frac{\kappa}{\kappa-\sigma\delta x})^2(1-N\delta x) + \frac{\sigma}{\sigma-\alpha}R(\sigma-\alpha, \frac{\sigma u}{\sigma-\alpha}, x)N\delta x \tag{27}$$

Subtract $R(\sigma,u,x)$ from both sides, divide by $\delta x$ and let $\delta x \to 0$, the result is

$$\frac{\partial R}{\partial x} - \frac{1}{\kappa}\frac{\partial}{\partial\sigma}(\sigma^2 R) + NR = \frac{N\sigma}{\sigma-\alpha}R(\sigma-\alpha, \frac{\sigma u}{\sigma-\alpha}, x) \tag{28}$$

If the beam stiffness is random with p.d.f. $S(\beta)$ then the result is

$$\frac{\partial R}{\partial x} - \frac{1}{\kappa}\frac{\partial}{\partial\sigma}(\sigma^2 R) + NR = N\sigma\int_0^\sigma R(\sigma-\xi, \frac{\sigma u}{\sigma-\xi}, x)S(\xi)\frac{d\xi}{\sigma-\xi} \tag{29}$$

These equations are to be solved subject to the usual normalising condition and the initial condition $R(\sigma,u,0) = 0$ for $\sigma u \neq F$. The equation for the marginal distribution $P(\sigma,x)$ may be obtained from (29) by integration over all values of $u$. The result is

$$\frac{\partial P}{\partial x} = \frac{1}{\kappa}\frac{\partial}{\partial\sigma}(\sigma^2 P) - NP + N\int_0^\sigma P(\sigma-\xi)S(\xi)d\xi \tag{30}$$

Appropriate conditions on the solution of this equation are $P(\sigma,0)$ given and $\int_0^\infty P(\sigma,x)d\sigma = 1$ . If the left end is attached to a spring of stiffness $\sigma^*$ , for example, the initial condition is $P(\sigma,0) = \delta(\sigma-\sigma^*)$ . No solutions of Eq. (30) have been obtained by the author except the one given in the last section for the semi-infinite structure when Eq. (30) reduces to Eq. (9) because $\frac{\partial P}{\partial x} = 0$ .

## Properties at an Interior Point

Let us now turn our attention to the problem of determining the joint p.d.f. of $\sigma$ , $u$ , and force $f$ (in the rod) at a point $x$ interior to a structure of finite length $L$ . This problem may be solved in terms of previously determined

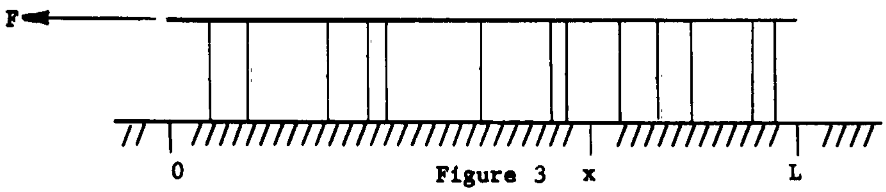


Figure 3

probability density functions if we make a cut in the structure at the point $x$ and then determine the forces and displacements necessary to restore continuity and equilibrium.

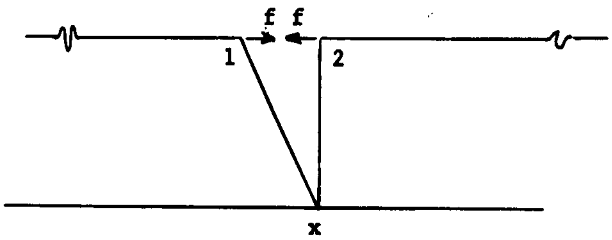


Figure 4

Figure 4 shows the cut structure near the point $x$. Let:

$\sigma_1$ = the stiffness at the point 1

$\sigma_2$ = the stiffness at the point 2

$\sigma$ = the stiffness of the continuous structure at $x$

$f$ = the force necessary to effect the closure which equals the force at $x$ on the continuous structure

$u_0$ = the gap between 1 and 2 before the two pieces of the structure are joined together

$u$ = the displacement at $x$ of the continuous structure (positive to the left)

$u_1$ = displacement of point 1 after the force $f$ is applied

$u_2$ = displacement of point 2 after the force $f$ is applied

We have

$$u_1 = u_0 - f/\sigma_1 \tag{31}$$

$$u_2 = f/\sigma_2 \tag{32}$$

The structure will be continuous and in equilibrium if $f$ and $u$ are determined from

$$u_1 = u_2 = u \tag{33}$$

This gives

$$f = \frac{\sigma_1\sigma_2 u_0}{\sigma_1+\sigma_2} \tag{34}$$

$$u = \frac{\sigma_1 u_0}{\sigma_1+\sigma_2} \tag{35}$$

The stiffness of the joined structure is obviously

$$\sigma = \sigma_1 + \sigma_2 \tag{36}$$

Now since $\sigma_1$ and $u_0$ are independent of $\sigma_2$ we may write

$$p(\sigma,u,f,x) = R(\sigma_1,u_0,x)P(\sigma_2,L-x)J\left(\frac{u_0,\sigma_1,\sigma_2}{\sigma,u,f}\right) \tag{37}$$

where p is the required p.d.f. and R and P have been previously defined. Equations (34), (35) and (36) may be solved for $\sigma_1$, $\sigma_2$ and $u_0$ with the result

$$\begin{aligned} u_0 &= \sigma u^2/(\sigma u-f) \\ \sigma_1 &= \sigma - f/u \\ \sigma_2 &= f/u \end{aligned} \tag{38}$$

From (38) we calculate J to be

$$J = \sigma/(\sigma u-f) \tag{39}$$

The result is thus

$$\begin{aligned} p(\sigma,u,f,x) &= R\left(\sigma - \frac{f}{u}, \frac{\sigma u^2}{\sigma u-f}, x\right)P\left(\frac{f}{u}, L-x\right)\frac{\sigma}{\sigma u-f} \quad ; \quad \sigma > \frac{f}{u} > 0 \\ &= 0 \qquad\qquad \frac{f}{u} < 0 \quad \text{or} \quad \sigma < \frac{f}{u} \end{aligned} \tag{40}$$

With a sufficient amount of effort one can derive an equation resembling a Fokker-Planck equation satisfied by p, but the results for L finite or infinite differ from each other. The author has been unable to arrive at these equations from first principles or to determine whether or not these equations together with certain boundary conditions constitute an appropriate formulation of the problem. The equation in question contains $\frac{\partial p}{\partial x}$ but not $\frac{\partial^2 p}{\partial x^2}$ and the association of such an equation with two point boundary conditions seems inappropriate. On the other hand there does not seem to be any reasonable second-order Fokker-Planck equation satisfied by p.

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